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ORIGIN OF THE NUCLEON ELECTROMAGNETIC FORM FACTORS DIPOLE FORMULA.

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Abstract

Starting with the VMD parametrization of the electric and magnetic nucleon form factors, which are saturated just by the ground state vector-mesons ρ, ω and ϕ, then aplying the strict OZI rule and the asymptotic behaviour of form factors as predicted by quark model of hadrons, the famous one parameter dipole formula is derived. By its comparison with space-like data up to $t = -5\ GeV^2$ the most optimal value of the parameter under consideration is determined. Finaly, charge and magnetization distributions in proton and neutron are predicted.

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The electromagnetic structure of nucleons is completely described by four scalar functions to be dependent on the square momentum transfer of the virtual photon $t = -q^2$. They can be chosen in the form of Sachs electric and magnetic form factors $G_E^p(t)$, $G_M^p(t)$, $G_E^n(t)$ and $G_M^n(t)$. Their behaviour is relatively complicated, especially in time-like region. In space-like region the behaviour of these form factors is well described by so-called dipole formula, which was founded in 1965 [1] before a discovery of the quark structure of hadrons determining asymptotic behaviours of the corresponding form factors. Generally, it has been found

$$G_E^p(t) \approx \frac{G_M^p(t)}{1+\mu_p} \approx \frac{G_M^n(t)}{\mu_n} \approx \frac{4m_n^2}{t}\frac{G_E^n(t)}{\mu_n} \approx \frac{1}{(1-\frac{t}{0.71})^2}, \tag{1}$$

at that time without any theoretical justification.

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Since that time theoretical knowledges of form factor behaviours have been improved. Vector-meson-dominance (VMD) model, based on a experimental fact of a creation of vector-meson resonances in e^+e^- annihilation into hadrons, has been created. In the framework of the perturbative QCD quark number dependent asymptotic behaviour of form factors has been revealed to be of the form

$$F \sim t^{1-n_q}. \tag{2}$$

In this paper using the abovementioned theoretical knowledges the nucleon form factors dipole formula is derived.

With this aim we start with the canonical VMD parametrization of the electric and magnetic nucleon form factors

$$\begin{aligned}
G_E^p(t) &= \frac{m_\varrho^2}{m_\varrho^2 - t}(f_{\varrho pp}^{(E)}/f_\varrho) + \frac{m_\omega^2}{m_\omega^2 - t}(f_{\omega pp}^{(E)}/f_\omega) + \frac{m_\phi^2}{m_\phi^2 - t}(f_{\phi pp}^{(E)}/f_\phi) \\
G_M^p(t) &= \frac{m_\varrho^2}{m_\varrho^2 - t}(f_{\varrho pp}^{(M)}/f_\varrho) + \frac{m_\omega^2}{m_\omega^2 - t}(f_{\omega pp}^{(M)}/f_\omega) + \frac{m_\phi^2}{m_\phi^2 - t}(f_{\phi pp}^{(M)}/f_\phi) \\
G_E^n(t) &= \frac{m_\varrho^2}{m_\varrho^2 - t}(f_{\varrho nn}^{(E)}/f_\varrho) + \frac{m_\omega^2}{m_\omega^2 - t}(f_{\omega nn}^{(E)}/f_\omega) + \frac{m_\phi^2}{m_\phi^2 - t}(f_{\phi nn}^{(E)}/f_\phi) \\
G_M^n(t) &= \frac{m_\varrho^2}{m_\varrho^2 - t}(f_{\varrho nn}^{(M)}/f_\varrho) + \frac{m_\omega^2}{m_\omega^2 - t}(f_{\omega nn}^{(M)}/f_\omega) + \frac{m_\phi^2}{m_\phi^2 - t}(f_{\phi nn}^{(M)}/f_\phi),
\end{aligned} \tag{3}$$

where $f_{vNN}^{(E,M)}$ are vector meson to nucleon coupling constants, f_v is universal vector-meson coupling constant and t is photon four-momentum tranfer squared. Taking into account OZI rule [2,3,4] strictly we require coupling constant of ϕ -meson to be zero.

Form factors G_E^p, G_M^p, G_E^n, G_M^n are normalized for the value $t = 0$ as follows

$$G_E^p(0) = 1; \;\; G_M^p(0) = 1 + \mu_p; \;\; G_E^n(0) = 0; \;\; G_M^n(0) = \mu_n, \tag{4}$$

where μ_p and μ_n are anomalous magnetic moments of proton and neutron, respectively.

For very large space-like t the asymptotic behaviour

$$G_{E,M}^{p,n} \sim t^{-2}, \tag{5}$$

is applied as predicted by the quark counting rules [5,6].

Requirements (4) and (5) lead to four systems of algebraic equations for coupling ratios

$$
\begin{aligned}
\text{I.}\qquad & (f^{(E)}_{\omega pp}/f_\omega) + (f^{(E)}_{\varrho pp}/f_\varrho) = 1 \\
& m^2_\omega (f^{(E)}_{\omega pp}/f_\omega) + m^2_\varrho (f^{(E)}_{\varrho pp}/f_\varrho) = 0 \\[2ex]
\text{II.}\qquad & (f^{(M)}_{\omega pp}/f_\omega) + (f^{(M)}_{\varrho pp}/f_\varrho) = 1 + \mu_p \\
& m^2_\omega (f^{(M)}_{\omega pp}/f_\omega) + m^2_\varrho (f^{(M)}_{\varrho pp}/f_\varrho) = 0 \\[2ex]
\text{III.}\qquad & (f^{(E)}_{\omega nn}/f_\omega) + (f^{(E)}_{\varrho nn}/f_\varrho) = 0 \\
& m^2_\omega (f^{(E)}_{\omega nn}/f_\omega) + m^2_\varrho (f^{(E)}_{\varrho nn}/f_\varrho) = 0 \\[2ex]
\text{IV.}\qquad & (f^{(M)}_{\omega nn}/f_\omega) + (f^{(M)}_{\varrho nn}/f_\varrho) = \mu_n \\
& m^2_\omega (f^{(M)}_{\omega nn}/f_\omega) + m^2_\varrho (f^{(M)}_{\varrho nn}/f_\varrho) = 0.
\end{aligned}
\tag{6}
$$

Their solutions take the form

$$
\begin{aligned}
\text{I.}\qquad & (f^{(E)}_{\omega pp}/f_\omega) = -\frac{m^2_\varrho}{m^2_\omega - m^2_\varrho} \\
& (f^{(E)}_{\varrho pp}/f_\varrho) = \frac{m^2_\omega}{m^2_\omega - m^2_\varrho} \\[2ex]
\text{II.}\qquad & (f^{(M)}_{\omega pp}/f_\omega) = -(1+\mu_p)\frac{m^2_\varrho}{m^2_\omega - m^2_\varrho} \\
& (f^{(M)}_{\varrho pp}/f_\varrho) = (1+\mu_p)\frac{m^2_\omega}{m^2_\omega - m^2_\varrho} \\[2ex]
\text{III.}\qquad & (f^{(E)}_{\omega nn}/f_\omega) = 0 \\
& (f^{(E)}_{\varrho nn}/f_\varrho) = 0 \\[2ex]
\text{IV.}\qquad & (f^{(M)}_{\omega nn}/f_\omega) = -\mu_n\frac{m^2_\varrho}{m^2_\omega - m^2_\varrho} \\
& (f^{(M)}_{\varrho nn}/f_\varrho) = \mu_n\frac{m^2_\omega}{m^2_\omega - m^2_\varrho},
\end{aligned}
\tag{7}
$$

which tranform form factors defined by (3) into relations

$$\begin{aligned}
G_E^p(t) &= \frac{1}{(1-\frac{t}{m_\omega^2})(1-\frac{t}{m_\varrho^2})} \\
G_M^p(t) &= (1+\mu_p)\frac{1}{(1-\frac{t}{m_\omega^2})(1-\frac{t}{m_\varrho^2})} \\
G_E^n(t) &= 0 \\
G_M^n(t) &= \mu_n \frac{1}{(1-\frac{t}{m_\omega^2})(1-\frac{t}{m_\varrho^2})}.
\end{aligned} \tag{8}$$

Masses of ω a ρ mesons are almost the same, therefore one can substitute these values in relations (8) by averaged mass m and finally for electric a magnetic nucleon form factors one gets expressions

$$\begin{aligned}
G_E^p(t) &= \frac{1}{(1-\frac{t}{m^2})^2} \\
G_M^p(t) &= (1+\mu_p)\frac{1}{(1-\frac{t}{m^2})^2} \\
G_E^n(t) &= 0 \\
G_M^n(t) &= \mu_n \frac{1}{(1-\frac{t}{m^2})^2}
\end{aligned} \tag{9}$$

consistent with the standard form of dipole formula besides the form factor $G_n^E(t)$, which is equal zero. This fact correspond with experimentally detected negligible values of $G_n^E(t)$. Dipole formula constant value is approximately $m^2 = 0.60\ GeV^2$.

Behaviour of form factor $G_E^n(t)$ can be determined including their additional properties. One can start from the identity

$$< r_n^2 >= 6\frac{dG_E^n(t)}{dt}\mid_{t=0} = 6\frac{dF_1^n(t)}{dt}\mid_{t=0} + \frac{3\mu_n}{2m_n^2}, \tag{10}$$

following from a decomposition of G_E^n into Dirac a Pauli form factors

$$G_E^n(t) = F_1^n(t) + \frac{t}{4m_n^2}F_2^n(t)$$

and normalization condition $F_2^n(0) = \mu_n$. According to the fact, that the last term value in (10) $-0.126\ fm^2$ is comparable with experimentally determined value of the neutron charge radius ($< r_n^2 >_{exp} = -0.119\ fm^2$ [7]), the term $\frac{dF_1^n(t)}{dt}\mid_{t=0}$ is almost zero and it can be

neglected. So, taking into account the later and the zero value of $G_E^n(t)$ at $t = 0$ one obtains a dipole formula for electric neutron form factor as follows

$$G_E^n(t) = \frac{\mu_n}{4m_n^2} t \frac{1}{(1 - \frac{t}{m^2})^2} \tag{11}$$

which describes data quite well.

If we define

$$G_D(t) = \frac{1}{(1 - \frac{t}{\lambda})^2}, \tag{12}$$

then the electric and magnetic nucleon form factors take the form

$$\begin{aligned} G_E^p(t) &= G_D(t) \\ G_M^p(t) &= (1 + \mu_p) G_D(t) \\ G_E^n(t) &= \frac{\mu_n}{4m_n^2} t G_D(t) \\ G_M^n(t) &= \mu_n G_D(t), \end{aligned} \tag{13}$$

identical with (1).

There are 433 experimental values of form factors in space-like region obtained from the elastic electron scattering on hydrogen and deuteron target, which have been analyzed by means of the relations (13) using program MINUIT.

One could expect, that two free parameter dependent dipole formula in (8) gives a better description of experimental data than canonical one free parameter dipole formula (13). However, the values of both free parameters in (8), determined in fitting procedure take almost the same values and therefore (8) is practically reduced to (9). In this case the dipole formula parameter takes the value $\lambda = 0.7134\ GeV^2$ ($\chi^2/ndf = 6.32$). Introducing non-zero value of $G_E^n(t)$ by (11) we obtain better description $\lambda = 0.7132\ GeV^2$ ($\chi^2/ndf = 5.92$).

In order to achieve acceptable value of χ^2/ndf it is necessary to reduce our fitting procedure to points up to $-5\ GeV^2$. χ^2/ndf takes a value 2.34 and $\lambda = 0.7263$. This results are graphically presented in Fig. 1. Strong enhancement of χ^2/ndf value suggests important deviation from dipole behaviour for low values of four-momentum transfer. Approximately 20% of total χ^2 is generated by $G_E^n(t)$. Relation (11) gives a worse description of experimental data, therefore it is impossible to decrease χ^2 value per point below limit $\chi^2/ndf = 2$.

Taking into account (13) one can predict charge and magnetization distribution of nucleons (Fig. 2). These results are compared with distributions given by unitary and analytic ten-resonance model presented by [8].

Although dipole formula gives a good description of all experimental values of electric and magnetic nucleon form factors, since this time it has not been known any theoretical motivation of these relations. Our paper present derivation of dipole formula including VMD model saturated by ground state of vector-meson ρ, ω and ϕ, OZI rule, normalization conditions and asymptotic behaviour of form factors given by quark model of hadrons.

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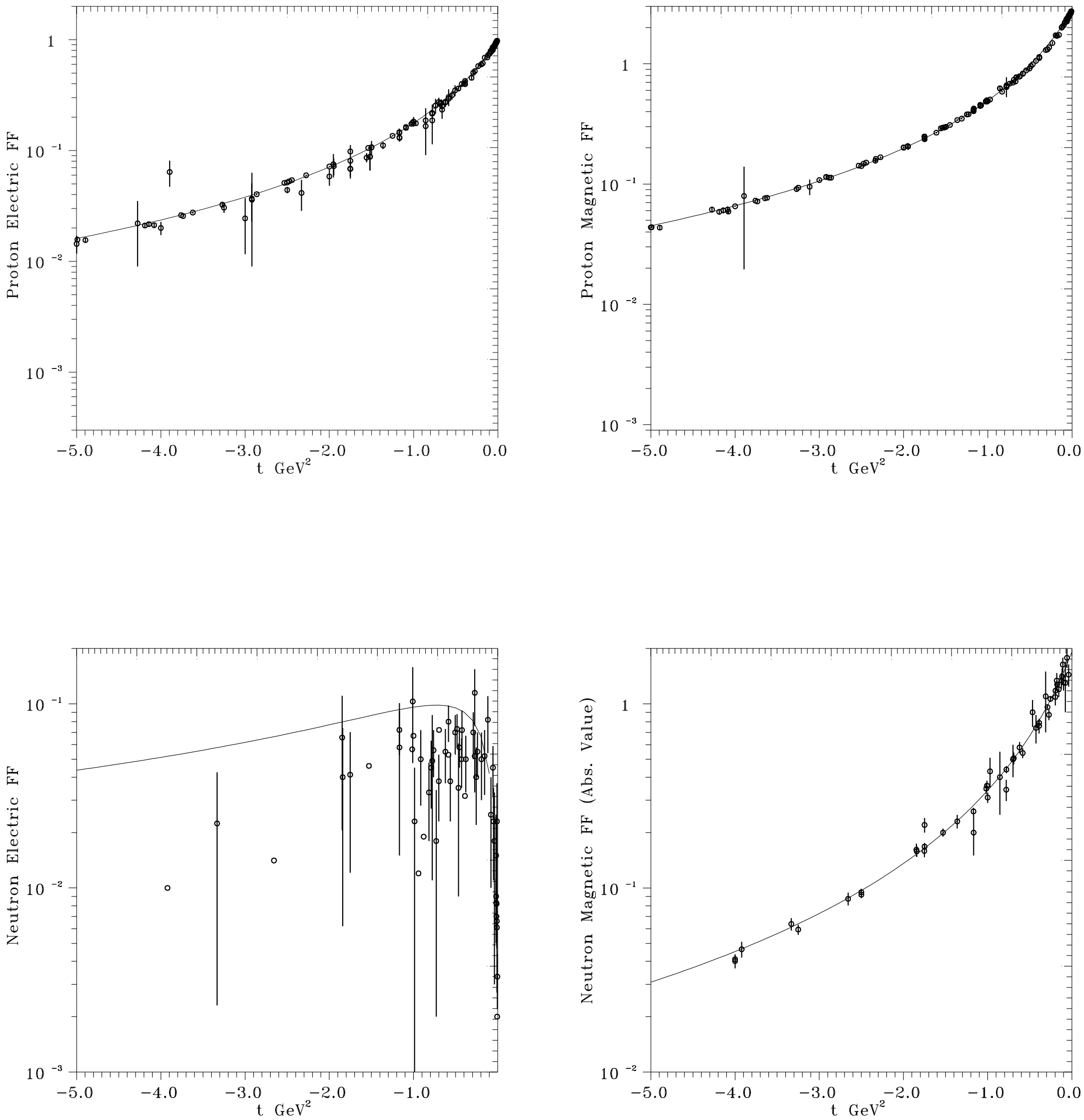

Figure 1: Behaviour of nucleon electric and magnetic form factors in space-like region given by dipole formula.

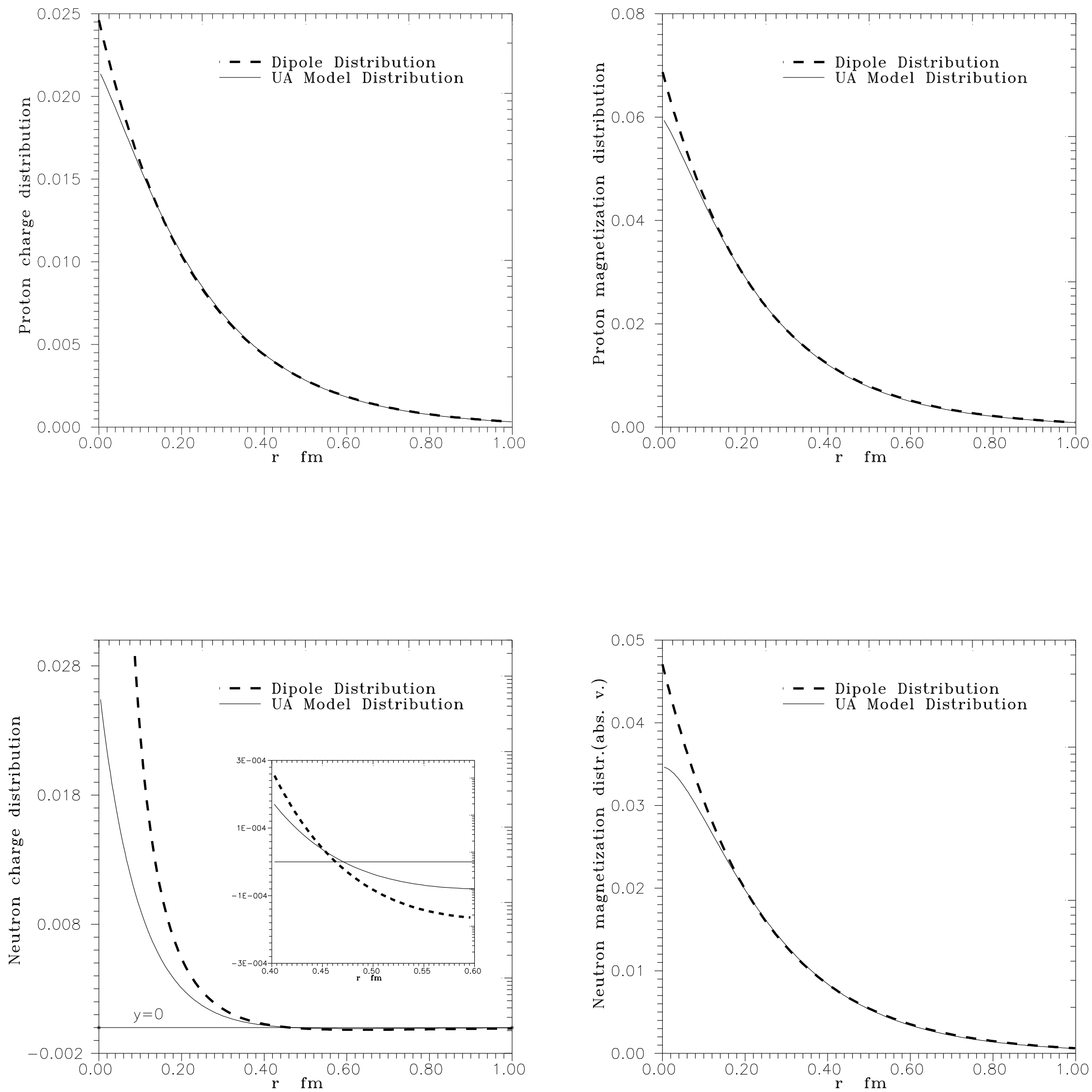

Figure 2: Predicted charge and magnetization distribution in proton and neutron.